# SPECIAL FEATURES OF FREE CONVECTION IN WATER AT TEMPERATURES BELOW 277° K

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An analysis of the parametric equations and of experimental investigations has shown the existence of a minimum in the heat transfer by free convection in water at temperatures below 277° K, for positive temperature heads, not exceeding 16°. A physical interpretation of the observed effect is given. The displacement of the heat-transfer minimum as a function of the temperature head is given.

Experimental investigations in moist air [6, 7] and in water [5], at temperatures of the latter close to 277° K, indicate that for specific conditions an "inversion of free convection" can occur, consisting of a change in the direction of motion of the convective flux. In particular, [5] describes experimental results of an investigation of the phenomenon of inversion with negative temperature heads (ice immersed in water).

The temperature head is understood to be the expression

$$\theta_s = t_s - t_w. \quad (1)$$

We shall call the temperature head positive when $t_s > t_w$ and negative when $t_s < t_w$.

It is interesting to note that the minimum heat transfer in this case occurs for a water temperature of 278° K (5° C). The authors of the article cited [5] explain the existence of the phenomenon of inversion of free convection in terms of the nature of the temperature dependence of the coefficient of thermal expansion of water, and introduce an empirical formula for calculating the coefficient, as applied to the case examined.

Since it is known that the coefficient of thermal expansion is a function of the change of density and is defined as

$$\beta = d\rho/\rho\, dt, \quad (2)$$

it is better to explain the inversion phenomenon by the special nature of the change of density of water with change of the water temperature. This approach leads to the conclusion that the inversion should be observed not only for negative, but also for small positive temperature heads. In addition, we should expect a displacement of the heat-transfer minimum along the temperature axis as a function of the value and sign of the temperature head. To verify the hypothesis, we analyzed parametric equations of the type Nu = $f$(Gr, Pr), or, what, amounts to the same thing, Nu = $f$(Ar, Pr).

Analysis shows that the parametric equations containing the determinant criterion Gr or Ar confirm the existence of a characteristic heat-transfer minimum and its displacement along the temperature axis as a function of the value and sign of the temperature head. By way of illustration, Fig. 1 shows graphs of the relation Nu = $f(T_w)$ for positive temperature heads of 2, 10, and 16°. The graphs were calculated from the equation [1]

$$\mathrm{Nu} = \mathrm{Nu}_{\min} + 0.6\,(\mathrm{Ar},\ \mathrm{Pr})^{0.25}, \quad (3)$$

where $\mathrm{Nu}_{\min}$ = 2 for a sphere and $\mathrm{Nu}_{\min}$ = 0.5 for a cylinder.

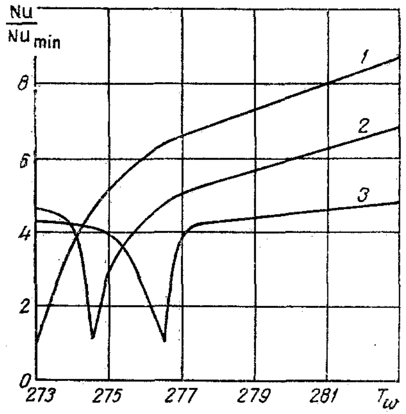


Fig. 1. Theoretical relation ($\mathrm{Nu}/\mathrm{Nu}_{\min}$ = $f(T_w)$ for temperature heads: 1) 16°; 2) 10°; 3) 2°.

This equation can be represented in expanded form as follows:

$$\mathrm{Nu} = \mathrm{Nu}_{\min} + 0.6\left(\frac{gl}{\nu_m^2}\,\mathrm{Pr}_m\frac{\rho_w - \rho_m}{\rho_w}\right)^{0.25}. \quad (4)$$

A calculation was carried out for a sphere of diameter 2 mm.

In choosing the physical parameters of the liquid, the characteristic temperature $t_m$ of the boundary layer was taken to be [1, 2]

$$t_m = \frac{t_w + t_s}{2}. \quad (5)$$

As can be seen from the graphs, the position of the minimum is displaced along the temperature axis as a function of the value of the temperature head. The law of displacement of the heat-transfer minimum can be obtained by making use of the analytical expression for the dependence of water density on temperature. As is known [4], for many liquids this dependence can be approximated very well by the empirical expression

$$\rho_t = \rho_0 + At + Bt^2 + Ct^3. \quad (6)$$

For water, Eq. (6) is somewhat simpler and has the form

$$\rho_t = \rho_0 + At - Bt^2. \quad (7)$$

It follows from [4] that the heat-transfer minimum will occur under the condition

$$\rho_w - \rho_m = 0. \tag{8}$$

Substituting the value of the density from (7) into (8), we obtain

$$At_w - Bt_w^2 - At_m + Bt_m^2 = 0. \tag{9}$$

On the basis of (1) and (5) we can write

$$t_m = \frac{\theta_s}{2} + t_w. \tag{10}$$

Substituting the value $t_m$ from Eq. (10) into Eq. (9) and carrying out the necessary transformations, we obtain

$$t_w + \frac{\theta_s}{4} - \frac{A}{2B} = 0. \tag{11}$$

Solving the equation obtained for $t_w$, we have

$$t_w = \frac{A}{2B} - \frac{\theta_s}{4}. \tag{12}$$

Since Eq. (7) is the equation of a parabola, the ratio of the constant coefficients A/2B is numerically equal to the temperature which corresponds to the maximum density, which for water is the temperature

$$t_{\max} = A/2B = 4^\circ \text{C}.$$

Going over to an absolute temperature scale, Eq. (12) takes the form

$$T_w = 277 - \frac{\theta_s}{4}. \tag{13}$$

Equations (12) and (13) are an analytical expression of the law of displacement of heat-transfer minimum as a function of the value and sign of the temperature head.

In some cases it is more convenient to operate directly with the temperature of the surface, and then Eq. (12), taking into account Eq. (1), can be written as

$$t_w = \frac{16 - t_s}{3}. \tag{14}$$

We shall apply the displacement law for water in the liquid phase, which for pure water corresponds to the condition $t_w \geq 0$, $t_s \geq 0$. The results obtained were checked experimentally in a temperature-controlled volume of still water by changing the temperature of the latter from 0 to 20° C. The quantitative measurements of the heat transfer for positive temperature heads were carried out by means of indirect heating [3] of semiconductor temperature sensors, based on thermoresistors of the types MT-54, KMT (MMT)-1, and others. The use of indirect heating, with the appropriate constructional form (surface heat source) and small measuring current passing through the thermoresistor, allowed us to obtain a thermoresistor temperature equal to that of the sensor surface, with sufficient experimental accuracy for this purpose. During the test the given temperature was maintained constant. The heat transfer was determined from the electrical power supplied to the heater winding of the temperature sensor.

The results of some tests for a sphere and a cylinder are shown in graphical form in Fig. 2. The graphs show the characteristic minimum and its displacement in accordance with the rules established above. Thus, an experimental confirmation was obtained of the existence of a heat-transfer minimum and its displacement as a function of the temperature head, for the case of low positive temperature heads.

The experimental results of [5] confirm the law established for the displacement of the minimum, for the case of negative temperature heads.

In fact, for the case of ice in water the temperature of the ice surface can be taken as equal to $t_s = 0$, which, from Eq. (14), corresponds to a heat-transfer minimum of

$$t_w = \frac{16 - 0}{3} = 5.3^\circ \text{C}.$$

The minimum obtained experimentally, as was mentioned above, is located at a temperature of 5° C. As can be seen, the discrepancy in the results is insignificant, and falls within our estimate of the limits of experimental accuracy.

On the basis of the experimental and theoretical data presented, it can be asserted that for low positive ($\theta_s = 16°$) and negative ($\theta_s = -5°$) temperature heads a minimum heat transfer in free convection occurs, the position of the minimum being displaced along the temperature axis in the range 0 to 5.3° C.

The minimum heat transfer observed for small positive temperature heads and the law of the displacement have not been explained, as far as we know, either in the Russian literature or abroad. However, the existence of the effect is undoubtedly of interest, both from the viewpoint of interpreting the phenomenon, and as regards certain applications. The observed effect can be given a simple physical interpretation. As has already been mentioned, the density of water varies according to a parabolic law, and has a maximum at 277° K (4° C) (Fig. 3). If we place a heated body in water at a temperature less than 277° K, then at some value of the temperature head the temperature $t_m$ in the boundary layer can prove to be greater than 277° K, whereupon the water density remote from the body, $\rho_w$, and in the boundary layer, $\rho_m$, will then be equal, $\rho_w = \rho_m$. Thus, in spite of the presence of a temperature gradient, free convection will be absent or will be practically minimal, and the heat transfer will then be due to thermal conductance of the water, $\text{Nu} \to \text{Nu}_{\min}$.

In conclusion we note that there is a region of applications where we are concerned with small temperature heads at water temperatures near 4° C, i.e., with conditions in which the observed effect occurs and must be taken into account. In particular, such conditions exist in measurements of small water flow velocities by means of thermal instruments. This minimum in the heat transfer can be used in cases when it is required to diminish or altogether to remove

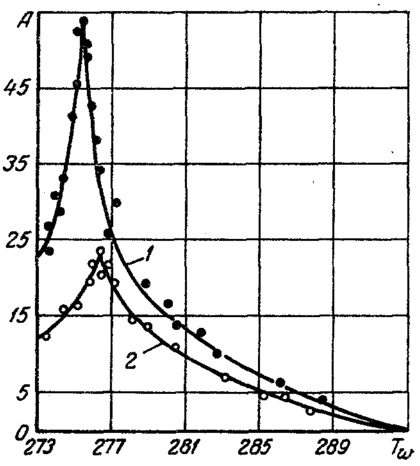


Fig. 2. Experimental dependence of $(Nu_{293} - Nu_t)/Nu_{293} \equiv A$ (%) as a function of $T_w$: 1) for sphere d = = 2 mm, $\vartheta_s = 7°$; 2) for cylinder d = 3.5 mm, L = 20 mm, $\vartheta_s = 2.1°$.

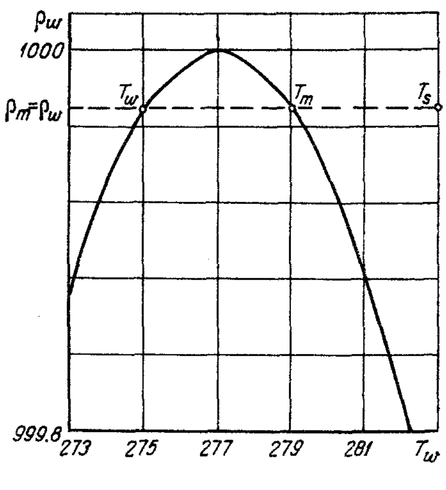


Fig. 3. Parabolic relation of water density as a function of temperature. The graph shows the equality of the densities $\rho_w = \rho_m$ for the case $T_w =$ = 275° K and $T_s$ = 283° K.

free convection in water near moderately heated bodies.

NOTATION

t is the temperature, °C; T is the absolute temperature, °K; $\rho$ is the density, kg/m$^3$; $\nu$ is the kinematic viscosity, m$^2$/sec; g is the acceleration due to gravity, m/sec$^2$; $l$ is the characteristic body dimension, m. The subscripts indicate the conditions to which the quantities pertain; m is the boundary layer, s is the body surface, and w is the medium remote from the body.

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2 March 1967

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